Reconnection in Magnetically Confined Fusion Plasmas

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Outline

Theory of reconnection in magnetically confined fusion plasmas
• recap of MHD equations and the role of resistivity
• formation of magnetic islands in tokamaks
• linear and nonlinear growth

Experimental examples of reconnection in tokamaks
• classical and neoclassical tearing modes
• rapid reconnection events: sawteeth
• others: MHD 'dynamo' through tearing modes
Theory of reconnection in magnetically confined fusion plasmas

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The one fluid MHD equations

\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (n\vec{v}) \]  

\text{equation of continuity}

\[ \rho \left( \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} \right) = -\nabla p + \vec{j} \times \vec{B} \]  

\text{force equation}

\[ \vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j} \]  

\text{Ohm’s law}

\[ \frac{d}{dt} \left( \frac{p}{\rho^\gamma} \right) = 0 \]  

\text{equation of state}

plus Maxwell’s equations for $E$ und $B$
Consider equilibrium (i.e. $dv/dt = 0$)

$$\nabla p = \vec{j} \times \vec{B} = \frac{1}{\mu_0} \left( \nabla \times \vec{B} \right) \times \vec{B}$$

So that two contributions to force balance can be identified:

$$\nabla_\perp \left( p + \frac{B^2}{2\mu_0} \right) + \frac{B^2}{\mu_0 R_c} \vec{e}_R = 0$$

- Magnetic pressure
- Field line tension
Consider equilibrium (i.e. $dv/dt = 0$)

$$\nabla p = \vec{j} \times \vec{B} = \frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B}$$

Use magnetic pressure to determine 'speed of sound' :

$$c_s = \sqrt{\gamma \frac{p}{\rho}} \approx \frac{B}{\sqrt{\mu_0 \rho}} \quad (\text{for } \beta = p_{\text{kin}}/p_{\text{mag}} << 1)$$

Alfven time scale:

$$\tau_A = \frac{L}{v_A} = \frac{L}{\frac{B}{\sqrt{\mu_0 \rho}}}$$

Fast (~ $\mu$s for 0.5 m) since mass is small in fusion experiments
MHD: consequences of Ohm's law

Consider equilibrium Ohm's law...

\[ \vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j} \]

...and analyse how magnetic field can change:

\[ \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{v} \times \vec{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B}) \]

\[ \Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \Delta \vec{B} \]

Flux conserving plasma motion (ideal MHD, \( \tau_A \))

Diffusive change of flux (resistive MHD, \( \tau_R \))
Consider equilibrium Ohm's law...

\[ \vec{E} = -\vec{v} \times \vec{B} + \frac{1}{\sigma} \vec{j} \]

...and analyse how magnetic field can change:

\[ \frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} = \nabla \times (\vec{v} \times \vec{B}) - \frac{1}{\mu_0 \sigma} \nabla \times (\nabla \times \vec{B}) \]

\[ \Rightarrow \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \frac{1}{\mu_0 \sigma} \Delta \vec{B} \]

Typical time scale of resistive MHD:

\[ \tau_R = \mu_0 \sigma L^2 \]

Since \( \sigma \) is large for a hot plasma, \( \tau_R \) is slow (~ sec for 0.5 m) – irrelevant?
Reconnection in a hot fusion plasma

Due to high electrical conductivity, magnetic flux is frozen into plasma

⇒ magnetic field lines and plasma move together

A change of magnetic topology is only possible through reconnection

• opposing field lines reconnect and form new topological objects
• requires finite resistivity in the reconnection region
Reconnection in a hot fusion plasma

Due to high electrical conductivity, magnetic flux is frozen into plasma

\[ \implies \text{magnetic field lines and plasma move together} \]

A change of magnetic topology is only possible through reconnection

- opposing field lines reconnect and form new topological objects
- requires finite resistivity in the reconnection region

Example: Coronal Mass Ejection (CME) from the sun
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Plasma can be confined in a magnetic field

Toroidal systems avoid end losses along magnetic field

- Need to twist field lines helically to compensate particle drifts
- Safety factor $q$: number of toroidal turns a field line completes for one poloidal turn
Plasma can be confined in a magnetic field.

'Tokamak': poloidal field component from strong toroidal plasma current

Due to radial variation of $T_e(r)$, there is also a radial variation $j(r)$

Free energy in the current density: the tokamak is prone to tearing modes
Reconnection in a tokamak

Typical $j$-profile

Corresponding $B_{pol}$-profile

Corresponding $q$-profile

Safety factor $q$: number of toroidal turns a field line completes for one poloidal turn

$$q = \frac{r B_{pol}}{R B_{tor}}$$

in cylindrical approximation

For centrally peaked $j(r)$, $B_{pol}$ increases weaker than linearly and hence, $q$ increases monotonically from the centre to the edge
Reconnection on ‘rational’ magnetic surfaces

Torus has double periodicity

- instabilities with poloidal and toroidal 'quantum numbers'

\[ m = 1 \]

\[ m = 2 \]

\[ m = 3 \]

‘Resonant surfaces‘ prone to instabilities with \( q = m/n \)
Helical field (i.e. 'poloidal' field relative to resonant surface) changes sign:

- reconnection of helical flux can form new topological objects - islands

Typical q-profile

$$B_{\text{hel}} = B_{\text{pol}} (1 - q / q_{\text{res}})$$

Corresponding $B_{\text{pol}}$

Corresponding $B_{\text{hel}}$
Helical field (i.e. 'poloidal' field relative to resonant surface) changes sign:

- reconnection of helical flux can form new topological objects - islands
Resistivity gives access to a new class of instabilities

Finite resistivity allows for changes in topology

\[ \sigma \to \infty: \text{ideal Kink} \]

\[ \sigma \neq \infty: \text{Tearing Mode} \]
Resistivity gives access to a new class of instabilities

'reconnection' of field lines at the 'X-point'

loss of thermal insulation across the island of width $W$ at the 'O-point'
Resistivity gives access to a new class of instabilities

'reconnection' of field lines at the 'X-point'
loss of thermal insulation across the island of width W at the 'O-point'
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• **linear and nonlinear growth**

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Deformation of flux surfaces opens up island of width $W$

- derive tearing mode equation for helical magnetic flux $\psi$
- may be considered as series of ideal MHD equilibria ($\nabla p = j \times B$)
- ideal MHD may be used everywhere except rational surface

Resistive MHD only important at the rational surface – timescale $\sim$ ms!
MHD description of tearing mode formation

Deformation of flux surfaces opens up island of width \( W \)

- Tearing Mode equation \( \nabla \rho = j \times B \) singular at resonant surface:
  - implies kink in magnetic flux \( \psi \), jump in \( B \)
  - \( \Rightarrow \) current sheet on the resonant surface

\[
\Delta \psi + \frac{\mu_0 d j(r)}{B_0 (1 - \frac{n}{m} q(r))} \psi = 0
\]

\( \psi(r) \): helical magnetic flux
\( j(r) \): current profile
\( q(r) \): field line helicity profile
\( m, n \): mode quantum numbers
Solution of tearing mode equation can be made continuous, but has a kink
• implied surface current will grow or decay depending on equilibrium $j(r)$
• the parameter defining stability is $\Delta' = \frac{(d\psi/dr)_{right} - (d\psi/dr)_{left}}{\psi}$
• if $\Delta' > 0$, tearing mode is linearly unstable – this is related to $\nabla j_{resonant\ surface}$
MHD description of tearing mode formation

(a) (2,1) mode unstable

\[ j_\phi(r) \sim (1 - (r/a)^2)^3 \]

(b) (3,1) mode stable

\[ j_\phi(r) \sim (1 - (r/a)^2)^3 \]

\[ \Delta'(W/a=0) = 6.4 > 0 \]

\[ \Delta'(W/a=0) = -5.5 < 0 \]
In the linear stage, the island growth is limited by resistive diffusion as well as the inertia of the plasma flowing into the island across the X-point.

The corresponding growth rate is

\[ \gamma = 0.55(\Delta' a)^{4/5} \tau_A^{-2/5} \tau_{res}^{-3/5} \]

However, once the island width is larger than the reonnection sheeth, the flow becomes unimportant and the growth enters into a nonlinear regime.
Consider various helical surface currents on resonant surface...

\[ B_\theta (r_s^+) - B_\theta (r_s^-) \propto \partial I = I_{Ohm} + I_{bs} + I_{extern} \]

\[ I_{Ohm} \propto j_{Ohm} W \sigma W d\psi / dt \propto \sigma W^2 dW / dt \]

inductive

\[ I_{bs} \propto j_{bs} W \propto -\nabla p / B_\theta W \]

pressure driven

\[ I_{extern} \]

externally driven

...leads to the so-called Rutherford equation (equivalent to Ohm's law)

\[ \tau_{res} dW / dt = a_1 \Delta' + a_2 \nabla p / W - a_3 I_{extern} / W^2 \]

where \( \Delta' = (B_\theta (r_s^+) - B_\theta (r_s^-)) / \psi \)
Due to the $1/R$ decay of a B-field in a torus, there is a magnetic mirror:

- particles with low $v_\parallel$ are trapped in this mirror, bounce back and forth
- poloidal projection of orbit resembles banana – ’banana orbit‘
For finite pressure gradient, there is a net current of trapped particles along field lines. Distortion of distribution function due to trapped particles leads to a net current of passing electrons.

Banana current $\rightarrow$ Bootstrap current
Tearing Modes – nonlinear growth

Interpretation of the different terms

\[ \tau_{res} \frac{dW}{dt} = a_1 \Delta' + a_2 \frac{\nabla p}{W} - a_3 \frac{I_{\text{extern}}}{W^2} \]

- for small \( \nabla p \), current gradient (\( \Delta' \)) dominates
  \( \Rightarrow \) 'classical Tearing Mode', current driven
- for larger \( \nabla p \), pressure gradient dominates:
  \( \Rightarrow \) 'neoclassical Tearing Mode', pressure driven
- adding an externally driven helical current can stabilise
‘Modified Rutherford equation’ for the temporal evolution of island width $W$:

$$\frac{\tau}{r_s} \frac{dW}{dt} = r_s \Delta' + 6.34 r_s \frac{\mu_0 L_q}{B_p} f_{GGJ} j_{bs} \frac{W}{W^2 + W_0^2} - 32 \frac{\mu_0 r_s L_q d}{B_p} j_{\text{extern}} \frac{1}{W^2}$$

- here, $j_{\text{extern}}$ is an externally generated *helical current* in the island
- effect of external current on stability via $\Delta'$ has been neglected
- intrinsic stabilisation at small island width ($W < W_0$)

Important: there is a number of additional terms that describe the stability at small island width. These are quite important for stability, but there is no agreed picture what the most important physics is…
NTMs = magnetic islands driven by lost bootstrap current inside island

- island shortcuts radial transport: flat pressure $\Rightarrow$ loss of bootstrap current
- at $\beta > \beta_{\text{marg}}$, finite seed island is sustained by plasma
- for negative $\Delta'$ (stable to classical tearing modes), NTM is metastable
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Magnetic islands are real ;-)
Here: try to increase the density by gas puff

- edge cooling increases $\nabla j(r)$, triggers $(3,1)$, then $(2,1)$ tearing mode
- related to the (edge) radiative instability discussed by X. Garbet yesterday
Interaction of islands can lead to sudden energy loss

W. Suttrop et al., *Nuclear Fusion* 1997

coupling between island chains (possibly stochastic regions)

⇒ sudden loss of heat insulation ('disruptive instability')
Interaction of islands can lead to sudden energy loss

Q. Yu et al., *Phys. Plasmas* 2006

'Intermediate' amplitude: Separated island chains

'Large' amplitude: Stochastisation

Coupling between island chains (possibly stochastic regions)

⇒ sudden loss of heat insulation ('disruptive instability')
Reminder: ideal MHD instabilities limit normalised pressure $\beta_{max} \sim l_p / (aB)$

'Troyon-Limit', leading to the definition of $\beta_N = \beta / (l/aB)$
Neoclassical tearing modes can occur well below ideal limit

- ‘practical $\beta$-limit’ in ITER standard scenario (ELMy H-mode)
- note: can also lead to disruptive termination (especially at low $q$)

NTMs = magnetic islands driven by lost bootstrap current inside island

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Metastable states of NTMs: stable at $W=0$, unstable at $W > W_{\text{seed}}$
Removal of magnetic islands by microwaves (ECCD)

Helical current can be driven by electron cyclotron resonance waves
Deposition controlled by local B-field ⇒ very good localisation
Feedback control of position possible via launch angle of ECCD beam
Adding a helical current \( \left( \frac{P_{\text{ECRH}}}{P_{\text{total}}} = 10\% \right) \) results in removal method has the potential for reactor applications…

N.B.: good agreement with Rutherford equation

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Sawtooth cycles as discovered on ST Tokamak

Repetitive increase and crash of central temperature

- increase slow, crash fast
- signal has the shape of a sawtooth
- note inversion of signal shape for $r > r_{inv}$

S. Von Goeler et al., *PRL* (1974)
Sawtooth cycles as seen in $T_e$ (JET)

Bigger machine – longer sawtooth period

- consistent with global current re-distribution time scale ($\sim r^2 T^{3/2}$)
Sawtooth crashes as seen in SXR (ASDEX Upgrade)

MHD signature of the mode preceding the crash is that of a (1,1) internal kink

Suggests that $q(0)$ falls below 1 due to current diffusion and this triggers the crash
Sawtooth cycles – effect on kinetic and q-profiles

Kadomtsev proposed a reconnection of (1,1) helical flux

- note: not a tearing instability, rather a 'driven reconnection' by the ideal (1,1) mode
- consistent with the experimental signature described so far
Kadomtsev proposed a reconnection of (1,1) helical flux

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Kadomtsev model for sawtooth crash

- Length $L$ (system size) $>>$ width $\delta \sim L/S^{1/2}$ \hspace{1cm} $S = \tau_R/\tau_A$
- Sweet-Parker Reconnection time $\tau_{SP} \sim \sqrt{\tau_R \tau_A} \sim S^{1/2} \tau_A$

Good for early experiments, but with increasing $S$, $\tau_{\text{crash}} \sim \text{const.}$ (!)
Kadomtsev model for sawtooth crash

Sweet Parker (MHD) reconnection time $\sim S^{1/2} \tau_A$ much longer than observed for large $S$ numbers

Solar flares: SP time $\sim 10^6$ s

Crash times up to a factor of 100 faster than predicted by Kadomtsev model

Slide stolen from S. Günter, EPS 2014
Current layer itself is not stable to start with!

For 2d reconnection without magnetic guide field:

- Critical aspect ratio $\sim 100$ (corresponding to $S_c \sim 10^4$) (e.g. Biskamp 1986)
- Linear growth rate and number of plasmoids increase with $S$ (e.g. Loureiro et al. 2007)
- Plasmoids significantly accelerate reconnection

Huang, Bhattacharjee 2010
Using two-fluid MHD provides further speed-up

Generalised Ohm’s law:

\[
\frac{d\Psi}{dt} = -\eta \mathbf{j} + \frac{1}{en_e} (\mathbf{j} \times \mathbf{B}) + \frac{1}{en_e} \nabla p_e - \frac{m_e}{e^2n_e} \frac{d\mathbf{j}}{dt}
\]

- Hall effect
- Whistler waves

- Electron pressure gradient
- Kinetic Alfvén waves

- Hall effect important in astrophysics, but currents perpendicular to magnetic fields are not relevant in reconnection in fusion plasmas

- Electron pressure gradient has qualitatively similar effect in fusion plasmas
Using two-fluid MHD provides further speed-up

Ions are decoupled from magnetic field on a scale $\rho_s$

$\Phi$

$\rho_s = \sqrt{T_e/m_i/\omega_{ci}}$

$\Psi$


Slide stolen from S. Günter, EPS 2014
Using two-fluid MHD provides further speed-up

S. Günter et al., Plasma Phys. Controlled Fusion 2014

Decoupling of ions from magnetic field leads to saturation of crash times and values comparable to those observed e.g. in ASDEX Upgrade.
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Advanced Scenarios for ITER: Improved H-mode

Example for Advanced Scenario:

- Improved H-mode, a.k.a. ‘hybrid’ scenario
- Confinement ($H_{98}$) and stability ($\beta_N$) improved w.r.t standard H-mode

Clue: subtle changes in the current profile have large impact on performance

$$q \approx \frac{r}{R} \frac{B_{tor}}{B_{pol}} \propto \frac{r^2}{R} \frac{B_{tor}}{I_p(r)}$$
Improved H-mode: temporal evolution of $q(0)$

Central safety factor clamped to $\sim 1$ even in the absence of sawteeth

- instead, strong (1,1) mode activity is observed
- note: similar story reported from DIII-D for (3,2) NTMs

ASDEX Upgrade – O. Gruber et al., Nucl. Fusion 1999
Sawtooth-free state with helical core

Poloidal velocity stream function (3D-2D)

Quasi-stationary state with helical core found in non-linear MHD simulations

Sawtooth-free state with helical core

Sawtooth free discharges due to (1,1) mode activity and associated plasma flows

- (1,1) mode driven by pressure gradient in weak shear region
- Associated with strong (poloidal) flows giving rise to a dynamo (loop) voltage

\[
\partial_t \Psi_0 = -R \Delta \eta J_{\phi,2D} - R \eta_{2D} \Delta J_{\phi} + R \hat{\phi} \cdot (v_1 \times B_1)
\]

Extra (dynamo) loop voltage counteracted by \( \Delta j_\phi \) – flattening of \( j_\phi(r) \)

Drive is the ideal instability of the ideal kink mode with weak magnetic shear

Note: dynamo quite common in Reversed Field Pinch (RFP)
Reconnection plays an important role in tokamak instabilities

- magnetic islands degrade confinement
- multiple magnetic islands can lead to disruptions

Magnetic islands in tokamaks are a testbed of our understanding of reconnection in hot ('collisionless') plasmas
- relatively well diagnosed (compared to Astrophysical Plasmas)

Single islands are often well described by simple one-fluid MHD
- (neo)classical tearing mode largely consistent with Rutherford equations

There are other phenomena that clearly need a description beyond simple one-fluid MHD
- fast reconnection as in sawteeth (or ELMs, that I have not talked about)